7.4 19. (1,1) $L=\int_1^4 \sqrt{1+\frac{1}{4x}}\,dx$

find the curve (equation $y=$ ____)

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad \frac{1}{4x}=\left(\frac{dy}{dx}\right)^2$$

$$\frac{dy}{dx}=\sqrt{\frac{1}{4x}}$$

$$y=\int\sqrt{\frac{1}{4x}}\,dx=\frac{1}{2}\int\sqrt{\frac{1}{x}}\,dx=\frac{1}{2}\int\frac{1}{\sqrt{x}}\,dx$$

$$y=\frac{1}{2}\int x^{-\frac{1}{2}}\,dx=\frac{1}{2}\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=\sqrt{x}+c$$

$1=\sqrt{1}+c \quad c=0$

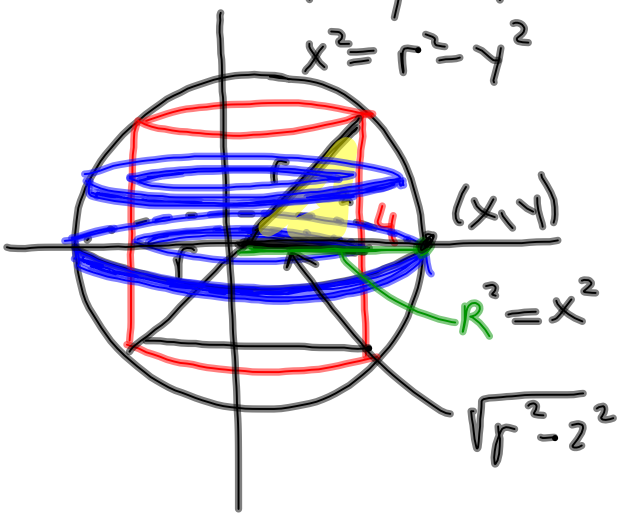
$$\boxed{y=\sqrt{x}}$$

Jan 7-10:01 AM

7.3 51

$x^2+y^2=r^2$

$x^2=r^2-y^2$



$$\int \pi R^2-\pi\left(\sqrt{r^2-2^2}\right)^2\,dy$$

$$\pi\int_{-2}^{2}(r^2-y^2)-(r^2-4)\,dy$$

$$\pi\int_{-2}^{2}\cancel{r^2}-y^2-\cancel{r^2}+4\,dy$$

Jan 7-10:11 AM

7.5a Applications from Science and Statistics

Work done by a variable force: $\boxed{W = \int_a^b F(x)\,dx}$

$W = F \cdot d \qquad F = \text{constant}$

$$W = \lim_{\Delta x \to 0} \sum F_i \, \Delta x = \int_a^b F \, dx$$

force equation

Dec 17-7:20 PM

A leaky ~~bucket~~ cauldron weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.

bucket: $W = 22\text{ N} \cdot 20\text{m} = 440\text{ N}\cdot\text{m} = 440\text{ J}$

water: $x=0 \quad F = 70\text{ N}$

$x=20 \quad F = 0\text{ N}$

$F = \left(\frac{70-0}{0-20}\right)(x-0)+70$

$F = -\frac{7}{2}x + 70$

$W = \int_0^{20} -\frac{7}{2}x + 70\, dx = 700\text{ J}$

rope:

$x=0 \quad F = (20\text{ m})(.4\ \frac{\text{N}}{\text{m}}) = 8\text{ N}$

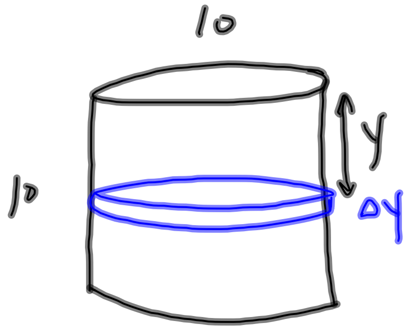
$x=20 \quad F = 0$

$W = \int_0^{20} -\frac{8}{20}(x-0)+8\, dx = 80\text{ J}$

Total: $440+700+80$ J

Dec 17-7:25 PM

How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?



work for 1 slice

Force of 1 slice (weight)

$$\Delta W = \Delta F \cdot y$$

$$\Delta w = \Delta V \, ft^3 \cdot 62.4 \frac{lb}{ft^3} \cdot y$$

volume of 1 slice

$$\Delta W = \pi \cdot 5^2 \, \Delta y \; 62.4 \; y$$

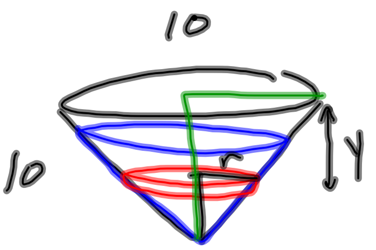
$$\Delta W = \pi \cdot 5^2 \cdot 62.4 \cdot y \cdot \Delta y$$

$$W = \int_0^{10} \pi \cdot 5^2 \cdot 62.4 \, y \, dy$$

245,044 ft-lbs

Dec 17-7:29 PM

A conical tank of height and diameter 10ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?



$$\frac{5}{10} = \frac{r}{10-y}$$

$$r = \frac{1}{2}(10-y)$$

$$\Delta W = \Delta F \cdot y$$

$$= \Delta V \cdot 57 \cdot y$$

$$= \pi r^2 \cdot \Delta y \cdot 57 \cdot y$$

$$= \pi \left[\tfrac{1}{2}(10-y)\right]^2 57 \, y \, \Delta y$$

$$\int_2^{10} \pi \left[\tfrac{1}{2}(10-y)\right]^2 \cdot 57 \cdot y \, dy$$

Dec 17-7:31 PM